satisfy the equation

$$dE/dt = -AE - D \tag{4}$$

Thus, the magnetic energy is diminished by the fluid motion of the form given by Eq. (1). It is to be noted that essentially the same result will be obtained if in Clark's work the sign of A is taken to be negative so that the flow direction is reversed.

Equation (3) can be reduced to the ordinary diffusion equation $\psi_\tau = \psi_{ss}$ by the transformation $\psi = H \exp(At)$, $\tau = (\eta/A)\{1 - \exp(-2At)\}$ and $s = x \exp(-At)$. Thus, the solution of Eq. (3) can be expressed in terms of the initial profile of the magnetic field $H_0(x)$ as follows:

$$H(x, t) = (2\pi\beta)^{-1/2}e^{-At} \int_{-\infty}^{\infty} H_0(\xi) \exp\frac{-(\xi - xe^{-At})^2}{2\beta} d\xi \tag{5}$$

where $\beta = (\eta/A)(1 - e^{-2At})$. Three examples have been worked out in detail for initial profiles:

$$H_0(x) = h_0 \exp(-x^2/2\delta^2) \tag{6}$$

$$H_0(x) = (h_0x/\delta) \exp(-x^2/2\delta^2) \tag{7}$$

$$H_0(x) = h_0(x/\delta)^2 \exp(-x^2/2\delta^2) \tag{8}$$

where h_0 and δ are constants. The solutions are, respectively,

$$H(x, t) = h_0\lambda^{-1/2} \exp\left\{-At - \frac{1}{2\lambda}\left(\frac{x}{\delta}\right)^2 e^{-2At}\right\} \tag{9}$$

$$H(x, t) = \left(\frac{h_0x}{\delta}\right)\lambda^{-3/2} \exp\left\{-2At - \frac{1}{2\lambda}\left(\frac{x}{\delta}\right)^2 e^{-2At}\right\} \tag{10}$$

$$H(x, t) = h_0\lambda^{-5/2}\left\{(\lambda - 1)\lambda e^{2At} + \frac{x^2}{\delta}\right\} \exp\left\{-3At - \frac{1}{2\lambda}\left(\frac{x}{\delta}\right)^2 e^{-2At}\right\} \tag{11}$$

where

$$\lambda = 1 + \epsilon(1 - e^{-2At}) \qquad \epsilon = \eta/A\delta^2$$

The total magnetic energy and the rate of dissipation are of the form, respectively,

$$E \sim \lambda^{-1/2}e^{-At} \qquad E \sim \lambda^{-3/2}e^{-At} \qquad E \sim \lambda^{-5/2}e^{-At}$$

$$D \sim \lambda^{-3/2}e^{-3At} \qquad D \sim \lambda^{-5/2}e^{-3At} \qquad D \sim \lambda^{-7/2}e^{-3At}$$

Since λ varies from 1 to $1 + \epsilon$ as t goes from 0 to infinity, the variation will be small for $\epsilon \ll 1$. The difference between these expressions for the three cases is, therefore, only slight.

Examination of the solutions obtained shows that the magnetic field intensity does, however, go up in value for $(x/\delta)^2$ sufficiently large, namely, for

$$x^2/\delta^2 > \lambda e^{2At} \tag{12}$$

Since the diminishing of field intensity at other values of x is large, the total magnetic energy actually goes down for all t. For the first example given by Clark [corresponding to Eq. (7)], it can be shown that

$$\begin{aligned}
\partial H/\partial t &\gtrless 0 && \text{when } (y^2/\delta) \lessgtr \epsilon + 2(1 - \epsilon)e^{-2At} - \epsilon^2(1 - \epsilon)^{-1}e^{2At} \quad t < t_E \\
&= 0 && \text{at } y = 0 \\
&< 0 && \text{at } y \neq 0 \quad \Big\}\ \text{when } t = t_E \\
&< 0 && \text{for all } y \quad \text{when } t > t_E
\end{aligned}$$

where $t_E = (1/2A)\ln\{2(1 - \epsilon)/\epsilon\}$.

The foregoing results show how the magnetic field energy will be modified by a stagnation flow. This modification depends on the orientation of the field with respect to the flow. In the three preceding examples for both symmetrical and antisymmetrical field profiles, the stagnation flow acts to lessen the total magnetic energy. But if the field direction is rotated 90° in the plane of the flow, amplification of the total magnetic energy will occur as shown by Clark.

References

[1] Parker, E. N., *Interplanetary Dynamical Processes* (Interscience Publishers, Inc., New York, 1963).

[2] Clark, A., Jr., "Production and dissipation of magnetic energy by differential fluid motions," Phys. Fluids **7**, 1299 (1964).

[3] Cowling, T. G., *Magnetohydrodynamics* (Interscience Publishers, Inc., New York, 1957).

Indirect Speed Measurement for Planetary Entry

M. G. Boobar* and B. A. McElhoe†

North American Aviation, Inc., Downey, Calif.

Nomenclature

A = area
C_D = drag coefficient
h = altitude
m = mass
$\dot{q}$ = heating rate/ft²
V = velocity
β = inverse scale height
θ = flight-path angle
ρ = density
ρ_0 = surface density

Subscripts

E = at entry
dep = at deployment

Introduction

Unmanned capsules, which enter planetary atmospheres at hypersonic velocities must be able to sense when the velocity has been reduced to a safe value if a parachute is to be deployed. Maximum time to sample the atmosphere and acquire experimental data on the atmospheric temperature, density, and composition profiles will be afforded by deploying a parachute at supersonic velocities. Two conditions must prevail at the time of parachute deployment in order to assure successful operation. First, the dynamic pressure must be low enough so that the parachute and shroud-line stresses do not exceed design limits. Second, if the parachute is deployed at supersonic velocities, aerodynamic heating must be below critical values. Unfortunately, direct measurement of the density and velocity behind the shock with sufficient accuracy is difficult, and inertial measurement or radar altimetry generally requires heavy, complex instrumentation.

This paper describes a simple technique for sensing when the velocity of a ballistic capsule entering a planetary atmosphere has decreased sufficiently to permit a parachute to be opened safely. The proposed technique does not require integration of the deceleration profile; rather, the velocity and density criteria are transformed to an acceleration criterion, which can be sensed easily with a simple accelerometer.

Received April 8, 1965.

* Project Manager, Space and Information Systems Division. Member AIAA.

† Project Staff Engineer, Space and Information Systems Division. Member AIAA.

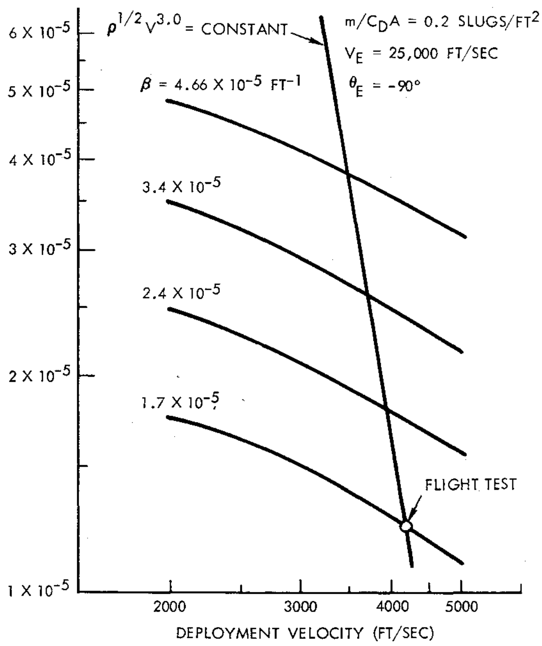

Fig. 1 Density-velocity paths for exponential atmosphere.

Entry into Exponential Atmospheres

Allen and Eggers[1] have shown that, for entry at steep angles into exponential atmospheres, the velocity varies as a function of altitude as follows:

$$V = V_E \exp[-(C_D A \rho_0 / 2\beta m \sin\theta_E) e^{-\beta h}] \tag{1}$$

assuming that the exponential atmosphere can be described by

$$\rho = \rho_0 e^{-\beta h} \tag{2}$$

The drag coefficient of the entry body is assumed constant, and the gravitational acceleration is considered negligible as compared to the aerodynamic deceleration. The relationship between density and velocity is derived from Eqs. (1) and (2) as

$$\rho = -2\beta(m/C_D A) \sin\theta_E \ln(V/V_E) \tag{3}$$

Employing the full range of values anticipated for the density gradient β of the Martian atmosphere,[2] Fig. 1 shows how the density varies as a function of velocity for entry at an angle of $-90°$ for a body having an $m/C_D A$ of 0.2 slugs/ft^2 at an entry velocity of 25,000 fps.

Recent flight tests with Cook Electric Company's Hyperflo parachute[3] have demonstrated successful deployment at Mach 4 velocities in the earth's atmosphere at an altitude of 120,000 ft; the corresponding density and velocity also are shown in Fig. 1. Scaling laws can be applied to estimate equivalent parachute heating in the Martian environment. The low Reynolds numbers encountered suggest that the flow on the skirt leading edge is laminar; the low velocities and small leading-edge radii preclude radiative heating. Leading-edge heating rates therefore were scaled as follows[1,4]:

$$\dot{q} \sim \rho^{0.5} V^n \tag{4}$$

where n is 3.0 to 3.5.

The line of equivalent heating (or velocity limit) given by Eq. (4) is shown in Fig. 2. As can be seen, the allowable deployment velocity decreases as the atmospheric density gradient increases. After selecting a deployment velocity based on the demonstrated operational capabilities of the parachute system and the highest density gradient β expected, a means of sensing when the vehicle has decelerated to a safe velocity is required.

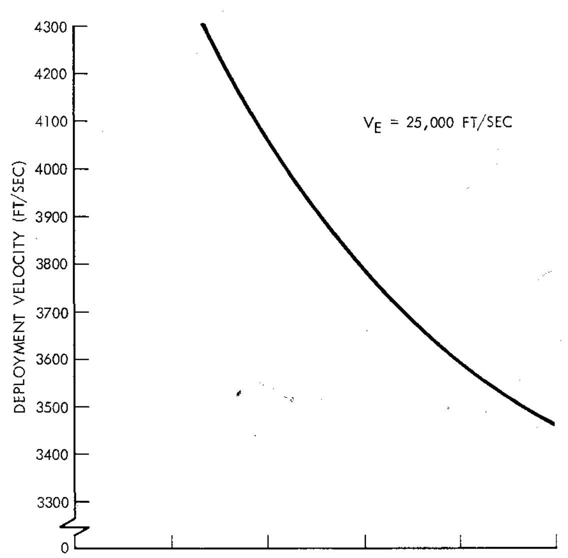

Fig. 2 Heating constraint on the maximum-allowable deployment velocity.

Velocity Sensing

Several good methods of measuring the velocity of a body undergoing atmospheric deceleration are available. These include a gimbal-mounted stable platform to support accelerometers decoupled from the vehicle oscillations or a strapped-down inertial system with a computer for resolving and integrating. These devices would measure the velocity change caused by drag with high accuracy at the expense of high weight and cost. More simple speed meters have been proposed that require only an integrating accelerometer and a pendulum to sense the direction of the resultant velocity vector. However, all of these techniques are sensitive to errors in the estimated atmospheric entry velocity; furthermore, good dynamic response and accuracy are required inasmuch as all of the errors are cumulative throughout the integration. The method proposed here does not require an accurate history of events; the decision is based only on instantaneous conditions.

Allen and Eggers[1] also have developed the following generalized relationships between the velocity and acceleration as a function of altitude for bodies of constant drag coefficient entering an exponential atmosphere:

$$V_{\Delta y}/V_E = \exp[-\tfrac{1}{2} e^{-\beta \Delta y}] \tag{5}$$

and

$$\frac{a_{\Delta y}}{a_{\max}} = \frac{(dV/dt)_{\Delta y}}{(dV/dt)_{\max}} = e^{-\beta \Delta y} \exp(1 - e^{-\beta \Delta y}) \tag{6}$$

where Δy is the altitude measured from the altitude at which maximum deceleration occurs.

Manipulating Eqs. (5) and (6), the velocity criterion for parachute deployment can be transformed into a criterion based on acceleration

$$\frac{a_{\text{dep}}}{a_{\max}} = -2e\left(\frac{V_{\text{dep}}}{V_E}\right)^2 \ln\left(\frac{V_{\text{dep}}}{V_E}\right) \tag{7}$$

A simple accelerometer aligned with the longitudinal axis of a nominally nonlifting entry body can be used to sense the maximum deceleration. The deceleration then is continuously compared with the stored maximum value. When the deceleration has reduced to a preset ratio, the parachute can be deployed with the assurance that the velocity has decreased to a safe value.

Several sources of error should be considered. Oscillations in angle of attack of nonlifting entry shapes are generally

Table 1 Deployment conditions in low-pressure Mars atmospheres

Atm	Entry angle, deg	Velocity, fps	Altitude, ft
G	−90	3750	37,000
...	−40	3150	51,000
...	−20	1320	81,000
H	−90	2800	30,000
I	−90	2450	38,000
J	−90	2250	62,000
...	−60	2430	69,000
...	−40	2440	81,000

very lightly damped; hence, the sensitive axis of a body-mounted accelerometer may oscillate about the drag vector. Studies using the Apollo shape have shown that, for angle-of-attack oscillations of 20° amplitude, a damped accelerometer could read about 2% low. The desired acceleration ratio should be biased to account for the possibility of this error, depending upon the accuracy with which the angle of attack at entry can be controlled, and, therefore, the oscillations, which can be expected during the time acceleration measurements are to be made. An uncertainty in entry velocity is also to be considered when establishing the desired acceleration ratio; however, this effect is reduced with the choice of deployment criterion discussed here.

The deployment criterion given in Eq. (7) is derived for any atmospheric model with constant scale height in the region of appreciable acceleration. If landers with a high ballistic coefficient are to enter with steep entry angles, the possibility of deploying the parachute in a troposphere must be considered. In this case, the relationship between acceleration and velocity must be determined from inspection of computed trajectory data for the specific atmospheres expected. Table 1 shows the velocity and altitude at which the acceleration is 12% of peak acceleration for entry into Kaplan's low-pressure model atmospheres[2] shown in Table 2. Note that, with this criterion, deployment does not necessarily occur at the highest altitude possible for the particular atmosphere encountered; rather, a minimum altitude for a model "extreme" atmosphere is assured. It should be noted also that these represent worst-case designs; in the event that the capsule enters at a low entry angle or encounters a more dense atmosphere, this criterion will deploy the parachute at a higher altitude and provide more time for measuring and transmitting scientific data.

Table 2 Characteristics of Kaplan atmospheres

Atmosphere	*G*	*H*	*I*	*J*
Surface pressure	11	11	15	30 mb
Stratosphere temp.	234	414	324	234 °R
Surface temp.	468	468	414	378 °R
Molecular wt	42.6	42.6	38.8	31.3
Tropopause alt	82,300	19,000	33,400	64,800 ft
Inverse scale height, β	4.506	2.546	2.963	3.308 10^{-5} ft
Surface density	4.21	4.21	5.90	10.42 10^{-5} slugs/ft^3
Artificial surface density	26.40	4.89	8.44	27.55 10^{-5} slugs/ft^3

References

[1] Allen, H. S. and Eggers, A. J., Jr., "A study of the motion and aerodynamic heating of missiles entering the earth's atmosphere at high supersonic speeds," NACA TN 4047 (1957).

[2] Framan, E., "Summary of Mars atmospheric models," Jet Propulsion Lab. Interoffice Memo. 313-1222 (November 1963).

[3] Nickel, W. E. and Sims, L. W., "A study and exploratory evaluation of deployable aerodynamic decelerators operating at high altitude and at high Mach numbers," Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio, FDL-TDR-64-35 (July 1964).

[4] Hoshizaki, H., "Heat transfer in planetary atmospheres at super satellite speeds," ARS J. **32**, 1544–1552 (1962).

Equilibrium Flow of an Ideal Dissociating Gas over a Cone

F. Edward Ehlers*

Boeing Scientific Research Laboratories, Seattle, Wash.

1. Introduction

FOR the ideal dissociating gas of Lighthill[1] under the assumption of thermodynamic equilibrium, the differential equations for the axially symmetric supersonic flow over a cone are derived, transformed, and simplified. Numerical results are given for flight in pure oxygen at a Mach number of 26.24 and for an ambient pressure corresponding to 100,000 ft altitude.

The equations of state of the ideal dissociating gas for the pressure p, density ρ, temperature T, enthalpy i, and fraction of dissociation α, as given by Lighthill,[1] are

$$p = \rho T(1 + \alpha) \tag{1}$$

$$i = (4 + \alpha)T + \alpha \tag{2}$$

$$\rho\alpha^2/(1 - \alpha) = e^{-1/T} \tag{3}$$

2. Equations of Motion for the Flow over a Cone

The equations of motion for the steady flow of a dissociating gas over a cone are given in Ref. 3. Eliminating the pressure by introducing the velocity of sound c, $c^2 = (dp/d\rho)_s$, the equations may be written in the form

$$d\sigma/d\omega = -v(u + v \cot\omega)/(v^2 - c^2) \tag{4}$$

$$dv/d\omega = c^2(u + v \cot\omega)/(v^2 - c^2) - u \tag{5}$$

$$du/d\omega = v \tag{6}$$

where u and v, the velocity components, and ω are described in Fig. 1; and $\sigma = \log\rho$. Differentiating the equilibrium relation, Eq. (3), logarithmically, yields

$$d\alpha = \alpha(1 - \alpha)dT/(2 - \alpha)T^2 - \alpha(1 - \alpha)d\rho/\rho(2 - \alpha) \tag{7}$$

This equation together with Eqs. (1) and (2) and the relation $di = dp/\rho$ yields

$$dT = T\varphi d\rho/\rho\psi \tag{8}$$

$$c^2 = (dp/d\rho)_s = T[2/(2 - \alpha) + \varphi^2/\psi] \tag{9}$$

where

$$\varphi = 1 + \alpha + \alpha(1 - \alpha)/(2 - \alpha)T$$
$$\psi = 3 + \alpha(1 - \alpha)/(2 - \alpha)T^2$$

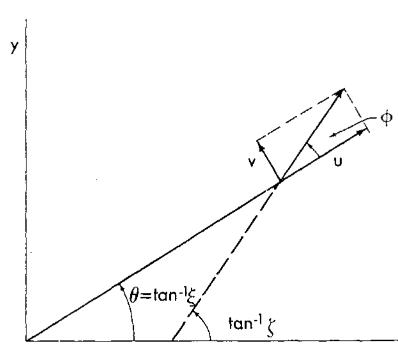

Fig. 1 Illustration of velocity vector and its components along rays through the cone vertex.

Received January 4, 1965; revision received April 14, 1965.

* Staff Member, Mathematics Research Laboratory. Associate Fellow Member AIAA.